

# Detection Guided Decision Feedback IIR Equalizer for sparse channels

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## Abstract

For the 4 wire-loop telephony circuit echo paths [1], mobile radio channels [2] and other channels, the impulse response of the channel has “sparsely active” characteristics. We consider a discrete NLMS adaptive IIR equalizer connected in cascade with the channel to compensate for the system degradation due to Intersymbol Interference (ISI) [3]. The conventional NLMS adaptive IIR equalizer adapts each and every tap at each sample interval. However, this approach suffers from the slow convergence problem. Motivated by the sparse channel characteristics, we investigate the use of active-parameter detection technique within the NLMS adapted IIR equalizer. The proposed detection technique is based on that employed for channel estimation applications in [4] [5]. The aim is to adapt only the active (or significant) taps of the equalizer. Improved convergence rates are achieved. The decision feedback technique is also incorporated in the IIR equalizer to cancel ISI [6]. Simulations of this newly proposed active-parameter detection guided IIR equalizer show the favourable accelerated convergence.

## 1. Introduction

For 4 wire-loop telephony circuit echo paths [1], mobile radio channels [2], room acoustic echo paths [7], echo paths in sonar [8] and geophysical applications [9] and other channels, it is relatively common that the impulse response of the channel has a “small” number of “active” (nonzero response) regions that are located in amongst relatively large inactive (zero response) regions. That is the multipaths occur in sparsely separated time delay clusters. To equalize such sparsely active channels, we consider a discrete NLMS adaptive IIR equalizer connected in cascade with the channel to compensate for the system degradation due to the Intersymbol Interference (ISI), as illustrated in Fig 1(a) & Fig1(b). The standard NLMS IIR equalization approach adapts each and every IIR tap  $w_i(k)$  during each sample interval. However, the standard NLMS adapted IIR equalizer suffers from the slow convergence problem when the required IIR equalizer has a long “tap” length. Motivated by the successful application of the detection guided channel estimator of [4] [5] to sparse channel estimation, we propose and investigate an active-parameter detection guided NLMS-IIR equalizer. The aim of the equalizer is to adapt only the detected active (or significant) taps of

the equalizer. Convergence rate improvement is achieved.

In this paper we incorporate the active-parameter detection technique and decision feed back equalization within the NLMS adapted IIR equalizer. Simulation results demonstrate the favourable fast convergence of this newly proposed IIR equalizer.

## 2. System Description

The configuration we consider in this paper is shown in figure 1.

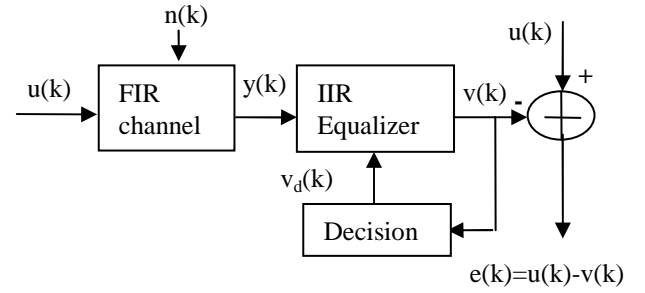


Figure 1a. System equalization configuration

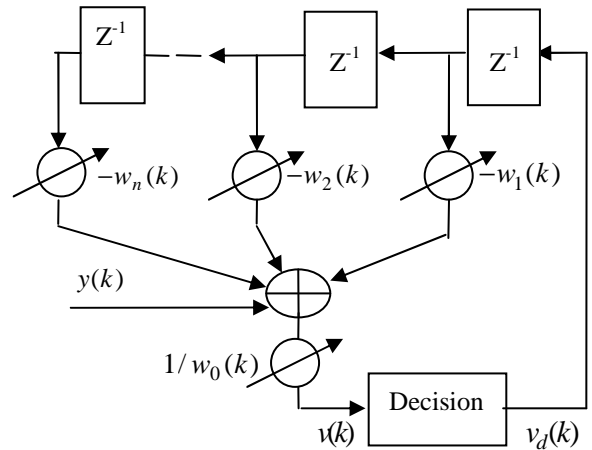


Figure 1b. Structure of IIR equalizer

### 2.1. Channel configuration

We assume the unknown channel is linear, time invariant which is adequately modeled by a discrete-time FIR filter  $\Theta = [\theta_0, \theta_1, \dots, \theta_n]^T$  with a maximum delay of  $n$  sample intervals:  $\Theta(k)[z^{-1}] = \theta_0(k) + \theta_1(k)z^{-1} + \dots + \theta_n(k)z^{-n}$ , where  $z^{-1}$  is the unit delay operator. We assume all

the signals are sampled. At sampling instant  $k$ ,  $U(k)=[u(k), u(k-1), \dots, u(k-n)]^T$  is the signal input vector to the unknown channel; an additive noise,  $n(k)$ , occurs within the unknown channel; and  $y(k)$  is the observed output from the unknown channel which is given by  $y(k)=U^T(k)\Theta(k)+n(k)$  where  $\Theta(k)=[\theta_0(k), \theta_1(k), \dots, \theta_n(k)]^T$ .

## 2.2. Equalizer configuration

The output of the decision feedback IIR equalizer is

$$v(k) = \frac{1}{w_0(k)} [y(k) - (w_1(k)v_d(k-1) + \dots + w_n(k)v_d(k-n))] \quad (1)$$

Where  $v_d(k) = \text{Decision}(v(k))$ .

For example, if  $u(k)$  is an antipodal binary sequence, then  $v_d(k) = \text{sign}(v(k))$ .

Equation (1) can be rewritten as

$$v(k) = a_0(k)y(k) + a_1(k)v_d(k-1) + \dots + a_n(k)v_d(k-n) \quad (2)$$

where  $a_0(k) = 1/w_0(k)$ ,  $a_1(k) = -w_1(k)/w_0(k)$ , ...,  $a_n(k) = -w_n(k)/w_0(k)$

If we let  $A(k)=[a_0(k), a_1(k), \dots, a_n(k)]^T$ ,  $S_d(k)=[y(k), v_d(k-1), \dots, v_d(k-n)]^T$ , then we get

$$v(k) = A(k)^T S_d(k) \quad (3)$$

The adaptive NLMS equalizer equation to update the parameter vector is:

$$A(k+1) = A(k) + \frac{\mu}{S_d^T(k)S_d(k) + \delta} S_d(k)e(k) \quad (4)$$

where  $e(k) = u(k) - v(k)$ , and where  $\mu$  and  $\delta$  are small positive constants.

In addition to the above, it is also assumed that

- (i) The elements of the input signal vector are samples of zero mean, bounded, wide sense stationary processes of variance  $\sigma_u^2$
- (ii) The noise signal is a zero mean, bounded, wide sense stationary white process of variance  $\sigma_s^2$ .
- (iii) The noise signal is uncorrelated with the input signal vector.

Furthermore, we assume the FIR channel  $\Theta(k)[z^{-1}]$  is sparsely active:

$$\Theta[z^{-1}] = \theta_0 + \theta_{t_2} z^{-t_2} + \dots + \theta_{t_m} z^{-t_m} \quad (5)$$

where  $m \ll n$ , and  $0 < t_2 < \dots < t_m \leq n$

The standard NLMS adaptive IIR equalizer adapts every coefficient  $a_i(k)$  [ $i=0, 1, \dots, n$ ] at each sample interval. However, this approach leads to slow convergence rates and poor tracking performance when the required IIR equalizer is 'long'. We propose to incorporate active-parameter detection within our NLMS adapted IIR equalizer, the aim of which is to adapt only the taps corresponding to the active or significant taps of the FIR channel.

## 2.3. Active tap definition

At sample instant  $k$ , an **active** tap or coefficient is defined as a tap corresponding to one of the  $m$  indices  $\{t_a\}_{a=1}^m$  of (5). Each of the remaining taps is defined as an **inactive** tap. Note: although not explicitly indicated in (5), the active tap indices  $t_a$ ;  $a = 1, 2, \dots, m$  and the number of active tap indices  $m$  may be time varying.

## 3. Activity detection guided NLMS IIR equalization

Based on the active-parameter detection criteria proposed in [4] [5] for channel estimation applications, we propose the following for our IIR channel equalization application.

The activity criterion within our detection guided IIR equalizer is derived from the structurally consistent least squares based cost function [11]:

$$V_{SCLS}(N) = V_{LS}(N) + m\sigma_u^2 \log N \quad (6)$$

where  $V_{LS}(N) = \sum_{k=1}^N [u(k) - v(k)]^2$ ;  $\sigma_u^2$  = variance of  $u(k)$ ;  $m$  = number of active taps;

For simplicity, we begin by assuming the covariance matrix  $C = \sum_k S_d(k)S_d(k)^T$  is diagonal. Then for sufficiently large  $N$  we may approximately replace equation (6) by:

$$\tilde{V}_{SCLS}(N) = \sum_{k=1}^N u^2(k) - \sum_{i=1}^m [X_j(N) - \sigma_u^2 \log N] \quad (7)$$

where  $X_j(N) = \frac{[\sum_{k=1}^N u(k)s_{d_j}(k)]^2}{\sum_{k=1}^N s_{d_j}^2(k)}$  and where  $s_{d_j}(k)$  is the  $j^{\text{th}}$  element of  $S_d(k)$ .

Hence, minimization of  $\tilde{V}_{SCLS}(N)$  is achieved by those equalizer taps  $a_j(k)$  for which at sample instant  $k$ ,

$$X_j(k) > T(k), \quad (8)$$

$$\text{Where } X_j(k) = \frac{[\sum_{i=1}^k s_{d_j}^{(i)} u(i)]^2}{\sum_{i=1}^k s_{d_j}^2(i)} \quad (9)$$

$$T(k) = \log(k)\sigma_u^2 \approx \frac{\log(k)}{k} \sum_{i=1}^k u^2(i) \quad (10)$$

In general, however the covariance matrix  $C$  is not diagonal, and as a result, coupling occurs between the taps within the numerator term  $\sum_{k=1}^N u(k)s_{d_j}(k)$ . This causes  $X_j(k)$  to be dependent not only on  $a_j$  but also on the neighbouring taps.

The following enhanced activity detection criteria are proposed to remove the coupling effects:

**Modifications 1:** Replace  $x_j(k)$  by:

$$\tilde{x}_j(k) = \frac{[\sum_{i=1}^k e(i)S_{dj}^{(i)} + a_j(i)S_{dj}^{2(i)}]^2}{\sum_{i=1}^k S_{dj}^{2(i)}} \quad (11)$$

Coupling effects between the neighbouring taps is reduced by the additional term  $a_j(i)S_{dj}^{2(i)} - v(i)S_{dj}^{(i)}$  in the numerator of  $\tilde{x}_j(k)$  [4] [5] [12].

**Modification 2:** Replace  $T(k)$  by:

$$\tilde{T}(k) = \frac{\log(k)}{k} \sum_{i=1}^k e(i)^2 \quad (12)$$

This is due to the realization that for *inactive taps*, the numerator term of  $\tilde{x}_j(k)$  is approximately:

$$N_j(k) \approx \sum_{i=1}^k e(i)S_{dj}^{(i)}, \quad j = \text{inactive tap index.}$$

Combining this with the LS theory on which the original activity criterion (8) is based, then suggests this modification [4] [5] [11].

**Modification 3:** Apply an exponentially decay forgetting operator  $F_k(i) = (1 - \gamma)^{k-i}$ ,  $0 < \gamma < 1$  within the summation terms of the activity criterion.

Modification 2 is theoretically correct only if  $e(k)$  is stationary. Clearly this is not the case. Modification 3 is included to reduce the effect of  $e(k)$  being non-stationary. Note, the inclusion of Modification 3 also improves the applicability of the detection guided equalizer to time varying systems.

Accordingly, the following detection guided NLMS adaptive algorithm is proposed for the IIR equalizer:

For each parameter index  $j$ ;

1. Label the tap index  $j$  to be a member of the active parameter set  $\{t_a\}_{a=1}^m$  at sample instant  $k$  if

$$\tilde{x}_j^F(k) > \tilde{T}^F(k), \quad (13)$$

Where

$$\tilde{x}_j^F(k) = \frac{\sum_{i=1}^k F_k(i)[e(i)S_{dj}^{(i)} + a_j(i)S_{dj}^{2(i)}]}{\sum_{i=1}^k F_k(i)S_{dj}^{2(i)}} \quad (14)$$

$$\tilde{T}^F(k) = \frac{\log(L^F(k))}{L^F(k)} \sum_{i=1}^k F_k(i)[e(i)]^2 \quad (15)$$

$$L^F(k) = \sum_{i=1}^k F_k(i) \quad (16)$$

And where  $F_k(i)$ , is the exponentially decaying operator:  $F_k(i) = (1 - \gamma)^{k-i}$   $0 < \gamma < 1$  and  $S_{dj}(i)$  is the  $j^{\text{th}}$  element of  $S_d(i)$ .

Otherwise, label the parameter index  $j$  as a member of the inactive parameter set.

2. Update the NLMS weight for each detected active parameter index  $t_a$ :

$$a_{t_a}^{(k+1)} = a_{t_a}^{(k)} + \frac{\mu}{\sum_{t_a} S_{d_{t_a}}^T(k)S_{d_{t_a}}(k) + \delta} S_{d_{t_a}}(k)e(k) \quad (17)$$

where  $\sum_{t_a}$  = summation over all detected active parameter indices.

3. Reset the NLMS weight to zero for each identified inactive parameter index.

## 4. Simulations

Simulations were carried out to investigate the performance (convergence rate, steady state error) of the following two equalizers:

- (A) Standard NLMS decision feedback IIR equalizer
- (B) Active-parameter detection guided NLMS Decision feedback IIR equalizer

### 4.1. Simulation conditions

- a. The systems considered were based on a channel vector  $\Theta$  which has 4 active taps and 12 inactive taps:  $[1, 0, 0, 0, -0.5, 0, 0, 0, 0.23, 0, 0, 0, -0.015, 0, 0, 0]$ . A plot of the vector is shown in figure 3.
- b. All poles of the channel lie inside the unit circle, which indicates that the system is stable.
- c.  $u(k)$  is the coloured channel input signal described by the model  $u(k) = g(k)/[1 - 0.8z^{-1}]$ , where  $g(k)$  is a discrete white Gaussian process with zero mean and unit variance
- d. Initial weighting vector  $A(0) = [1, \text{zeros}(15)]$
- e. noise signal  $n(k)$  = zero mean Gaussian process with variance=0.001
- f. Regularisation parameter  $\delta = 0.1$
- g. Squared channel equalization error  $\|\theta - w\|^2$  is plotted to compare the convergence rate where  $W = [w_0, w_1, \dots, w_n]$ . All plots are the average of 10 similar simulations.

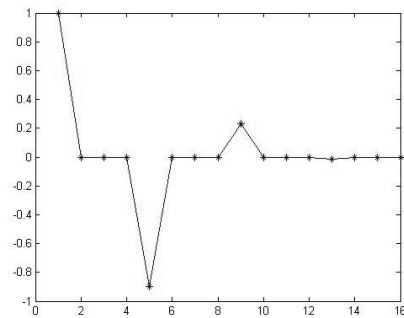


Figure. 2 Channel impulse response

## 5. Results and analysis

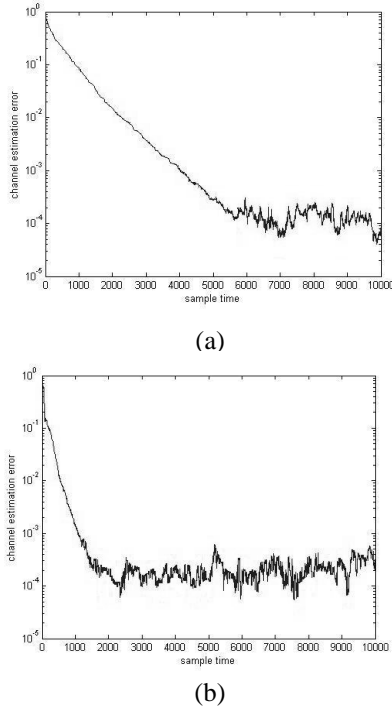
### 5.1. Simulation 1

Simulations between the standard NLMS decision feedback IIR equalizer (a) and the active-parameter detection guided NLMS decision feedback IIR equalizer (b) with different adaptive step size  $\mu = 0.03$  &  $\mu = 0.05$

The results of these are shown in Figure 3 & Figure 4 respectively.

**Part a** for the smaller step-size  $\mu = 0.03$  (In this case, the forgetting parameter  $\gamma$  is chosen to be 0.005) :

- Active-parameter detection guided decision feedback IIR equalizer (b) provides significantly improved convergence rate than the standard NLMS decision feedback IIR equalizer (a). This is due to the significant reduction in the number of taps involved in the weights adaptation.
- From figure.3, we can see that the two IIR equalizers almost have essentially the same steady state error. This indicates that the fast convergence in the active-parameter detection technique is not achieved at the cost of poorer steady state error performance.

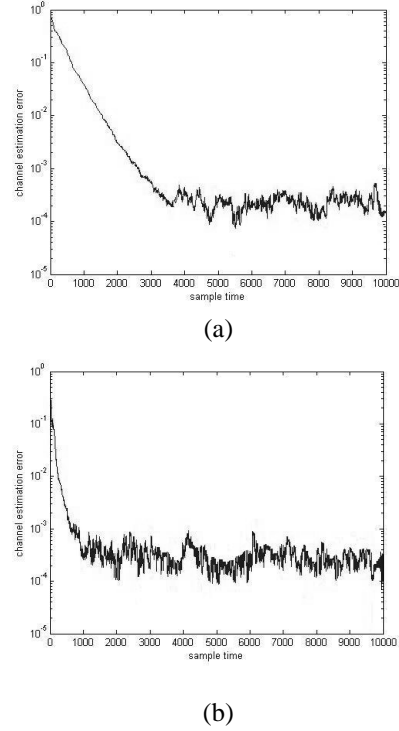


**Figure.3 Simulations for IIR equalizer (a) and IIR equalizer (b) with  $\mu = 0.03$**

**Part b** for the larger step-size  $\mu = 0.05$  (Again, the forgetting parameter  $\gamma$  is chosen to be 0.005) :

- Active-parameter detection guided IIR equalizer (b) shows significantly faster convergence than the standard NLMS IIR equalizer (a).

- As  $\mu$  increases, the convergence rate of the active-parameter detection guided FIR equalizer (b) increases, but on the other hand, the increase in  $\mu$  leads to higher steady state error. This is due to fact that the theoretical unbiased NLMS steady state error is being linearly proportional to the  $\mu$  value [12], hence, with an increase in the  $\mu$  value, the steady state error will increase accordingly. Note that this increase in steady state error occurs for both equalizers.



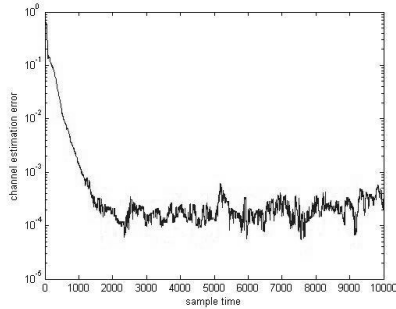
**Figure. 4 Simulations for IIR equalizer (a) and IIR equalizer (b) with  $\mu = 0.05$**

### 5.2. Simulation 2

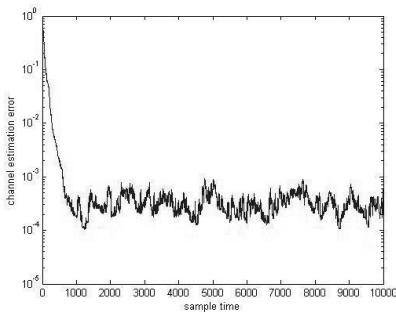
Simulations of the active-parameter detection guided NLMS decision feedback IIR equalizer with different forgetting parameter  $\gamma = 0.005$  &  $\gamma = 0.03$  (In this case, the step size  $\mu$  is chosen to be 0.03) .The results are shown in Figure 5 & Figure 6 respectively.

- As  $\gamma$  increases, the detection guided FIR equalizer is able to pick up less of the non-active taps, which results in faster convergence.
- With larger  $\gamma$  value, some of the active taps may be failed to be detected, which leads to poorer steady state error performance.
- For the simulation with  $\gamma = 0.005$ , all four of the active taps were detected; but for the simulation with  $\gamma = 0.03$  the detection guided decision feedback IIR equalizer failed to detect the smallest of the four active taps, whose amplitude is only 0.015. The failure to detect the fourth active taps

leads to an increase of  $0.015^2 = 2.25 \times 10^{-4}$  in the steady state error, which agrees with that of figure 5 and figure 6.



**Figure 5. Simulations for active-parameter detection guided IIR equalizer (b) with  $\gamma = 0.005$**



**Figure 6. Simulations for active-parameter detection guided IIR equalizer (b) with  $\gamma = 0.03$**

## 6. Conclusion

For equalization of sparsely active FIR channels, we have considered the adaptive NLMS IIR equalisation method. In this paper, we propose an active (or significant) parameter detection guided NLMS-IIR equalizer which has significant convergence rate advantage over the standard NLMS-IIR equalizer. Simulation results confirmed this favourable performance.

## 7. References

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